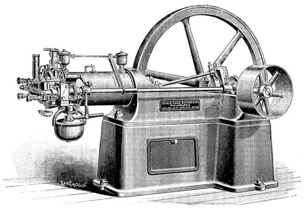

Petrol engine (ca. 1880).

Exercise 12.1: Combustion motor (4P)

Estimate the thermodynamic cycle in a petrol engine assuming an ideal gas in a quasi-static approximation.

$A \to B$:	Adiabatic compression from volume V_A to volume V_B.
$B \to C$:	The explosion increases the pressure by Δp.
$C \to D$:	Adiabatic expansion from volume V_B to volume V_A.
$D \to A$:	Cooling of the gas at constant volume.

An ideal gas obeys the state equation $E = \frac{3}{2}pV = \frac{3}{2}Nk_BT$.
An ideal gas expands/compresses adiabatically along the adiabatic lines $pV^{5/3} = const.$

(a) Sketch the cyclic process in a (p, V) diagram as well as in a (H, V) diagram. (1P)

(b) Compute the work during adiabatic compression and expansion. (1P)

(c) Show that the efficiency of the machine in a whole cycle is given by (2P)

$$\eta = \frac{\Delta W}{\Delta Q} = 1 - \left(\frac{V_B}{V_A}\right)^{\frac{2}{3}}$$

Exercise 12.2: Quantum particle in a 1D potential well (8P)

Let us consider a single quantum-mechanical particle in a one-dimensional infinite potential well of length L in contact with a thermal reservoir at temperature T.

(a) Compute the energy eigenvalues and write down the partition sum. (1P)

(b) Show that in the low-temperature limit $T \to 0$ the logarithm of the partition sum is given by leading and next-to-leading order by (2P)

$$\ln Z(\beta, L) = -\frac{\beta\gamma}{L^2} + e^{-3\frac{\beta\gamma}{L^2}} + \mathcal{O}\left(e^{-8\frac{\beta\gamma}{L^2}}\right) \quad \text{where} \quad \gamma = \frac{\pi^2\hbar^2}{2m}.$$

(c) Compute the average energy E, the heat capacity C, and the pressure P in the limit of low temperatures by calculating the corresponding derivatives of $\ln Z$. (1P)

(d) Why is $P > 0$ in the limit $T \to 0$? (1P)

(e) Approximate the partition sum in the high-temperature limit $T \to \infty$. (2P)

(f) Compute E, C, and P in the limit of high temperatures. (1P)

(Σ = 12P)

To be handed in on Monday, January 20, at the beginning of the lecture or directly to your tutor.