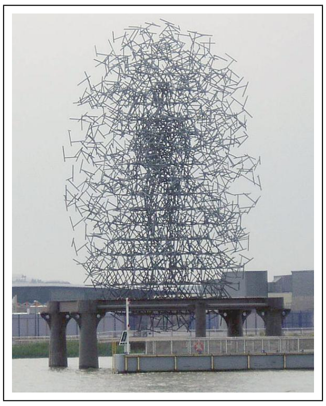
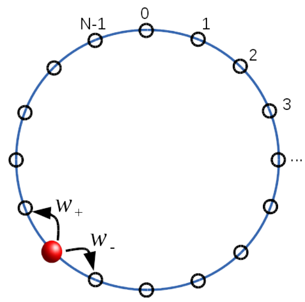


Random Sculpture by Antony Gormley
(London) [Andy Roberts, Wikimedia]

Exercise 8.1: Random walk on a ring **(12P)**

Consider a ring with N sites enumerated by $i = 0, 1, \ldots, N-1$. A particle (the red bullet) moves on the ring by jumping randomly to the neighboring sites. The random walk of the particle is biased, i.e., it hops to the right and to the left with generally different constant rates w_+ and w_- (see figure).



(a) Compute the matrix elements $\mathcal{L}_{ij}$ of the time evolution operator $\mathcal{L}$ for general N (using Kronecker-δ's). Write down the explicit matrix for $N = 4$ and check whether the columns add up to 0. (2P)

(b) Guess the stationary state $|P_{stat}\rangle = |P_\infty\rangle$ for general N guided by physical arguments. Verify your result and compute the entropy of the stationary state. (2P)

(c) Because of translational invariance the eigenvectors $|\psi^{(k)}\rangle$ of $\mathcal{L}$ can be determined for general N by means of an exponential ansatz

$$\psi_j^{(k)} = \langle j|\psi^{(k)}\rangle = e^{ikj} \,.$$

How are the values of k constrained by the periodicity of the ring and what is the allowed range of k in which all solutions are unique? (2P)

(d) Compute the corresponding eigenvalues λ_k and their real part $Re[\lambda_k]$. (2P)

(e) Determine the dominating relaxation time τ_{max}. (1P)

(f) How does the largest relaxation time scale with N if N is very large? Why is the result reasonable with respect to the central limit theorem? (2P)

(g) Under which conditions does the stationary state obey detailed balance? (1P)

Note: In all parts of the exercise, if not stated otherwise, N is assumed to be arbitrary.

(Σ = 12P)

To be handed in on Monday, December 09, at the beginning of the lecture (PG1 directly to Pascal).