Statistical Physics & Thermodynamics

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Exercise 10.1: Equation of State (4P)

Let us consider a thermodynamical system for which the entropy is given by

$$H(E, V, N) = (EVN)^\alpha$$

with an exponent $\alpha > 0$.

(a) Determine the temperature, the pressure, and the chemical potential. (1P)

(b) For which value of α are the results obtained in (a) physically reasonable? (1P)

(c) With (b), compute the chemical potential as a function of (T, V, N). (1P)

(d) With (b), compute the pressure as a function of (T, V, N). Sketch qualitatively an isothermal line (a line of constant T, N) in a P-V diagram. (1P)

Exercise 10.2: Expansion coefficient and compressibility (2P)

Express the *isobaric expansion coefficient* $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P,N}$ as well as the the *isothermal compressibility* $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N}$ as second derivatives of the appropriate thermodynamic potential.

Exercise 10.3: Free energy of a perturbed system (6P)

A classical system in thermal equilibrium with a heat bath at temperature T is described by an energy function $E^{(0)} : \Omega^{\text{sys}} \to \mathbb{R} : s \to E_s^{(0)}$. Let $Z^{(0)} = \sum_s e^{-\beta E_s^{(0)}}$ be the partition sum of the system.

(a) Assume that the energy function is perturbed by $E_s^{(0)} \to E_s = E_s^{(0)} + \lambda E_s^{(1)}$ with $\lambda \ll 1$. Compute the corresponding partition sum $Z(\lambda)$ as a power series in λ. (2P)

(b) What is the mathematical meaning of $Z(\lambda)$? Hint: Try to compute $\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} Z(\lambda)\big|_{\lambda=0}$ (1P)

(c) Prove the general statement that in the canonical ensemble the free energy $F = E - TH$ (more precisely: $\langle F \rangle = \langle E \rangle - T\langle H \rangle$) is given by $F = -T \ln Z$. (1P)

(d) Apply (c) to (a),(b) in order to compute $F(\lambda)$ as a power series in λ up to second order. What is the mathematical meaning of $F(\lambda)$? (2P)

(Σ = 12P)

To be handed in either on Monday, December 23 at the beginning of the lecture, or until Tuesday, January 7th, directly to your tutor.