Statistics of misprints [Wikimedia]

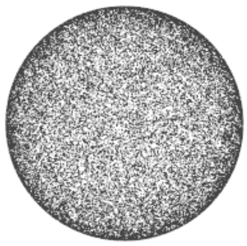
## Exercise 5.1: Statistics of misprints (2P)

In a printing company, the error probability for a typo is $q = 10^{-6}$. The typos occur completely uncorrelated and the probability is the same for all characters.

(a) Let $n$ be the number of correct characters between two subsequent typos. Compute the probability distribution $P(n)$ in an infinitely long text and check if it is properly normalized. (1P)

(b) Determine the mean value and the variance of this distribution. (1P)

## Exercise 5.2: Uniform probability density on a unit sphere (4P)



Consider a unit sphere with standard spherical coordinates $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. The figure shows uniformly distributed random points on the surface.

(a) Calculate the normalized probability density $p(\theta, \phi)$ which is uniform (i.e. constant per unit area) on the surface of the sphere. (1P)

(b) Show that this probability density factorizes. (1P)

(c) Let $X_1$ and $X_2$ be two uncorrelated continuous random variables with a uniform distribution between 0 and 1 (e.g. $x_1, x_2$ can be drawn from a standard random number generator). Transform $x_1, x_2 \in [0, 1]$ to spherical coordinates $\theta, \phi$ in such a way that the resulting probability on the unit sphere is uniform. (2P)

## Exercise 5.3: Entropy of spin-1 particles (6P)

Consider $N$ distinguishable classical spin-1 particles where each of them is in one of three spin states $S_z = 0, \pm 1$. Let $n_0, n_+$ and $n_-$ (with $N = n_0 + n_+ + n_-$) be the corresponding occupation numbers.

(a) Calculate the entropy for given $n_0, n_+, n_-$ (with $N = n_0 + n_- + n_+$). (1P)

(b) Apply Stirling's formula $\ln(m!) = m \ln m - m$ to derive an approximate expression for the entropy for large $N$. Determine the maximum of the approximated entropy and the corresponding values of the occupation numbers. (2P)

*Please turn over* ⇒

(c) Why is the approximated entropy determined in (b) too large? (1P)

(d) Show that the improved Stirling formula $\ln(m!) = m \ln m - m + \frac{1}{2} \ln(2\pi m)$ gives in fact an additional correction that decreases the value of the entropy determined in (b). (1P)

(e) Impose the additional constraint $n_- = 2n_+$ and determine the occupation numbers for which the approximated entropy is maximal. (1P)

**($\Sigma$ = 12P)**

To be handed in on Monday, November 18, at the beginning of the lecture (PG1 directly to Pascal).