Statistical Physics & Thermodynamics

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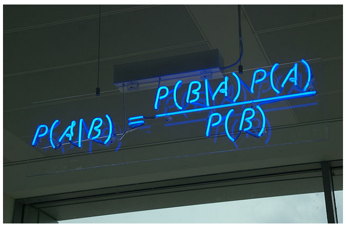

Bayes formula in a cafeteria [Wikimedia]

Exercise 2.1: Fixed points of a differential equation (3P)

(a) Determine the fixed points of the differential equation $\dot{x}(t) = ax(t)\big(1 - x^2(t)\big)$ and their stability. (2P)

(b) Let $|x(0)| \ll 1$ be very small. Compute the typical time that the system needs to 'switch' to a stable fixed point. (1P)

Exercise 2.2: Probability density of the logistic map (6P)

Let us consider the logistic map

$$x_{n+1} = f(x_n) \qquad \text{with} \qquad f(x) = 4x(1-x)$$

The purpose of this exercise is to show that this map is fully chaotic and that the probability to find x in a certain infinitesimal interval can be computed analytically.

(a) Prove that the Dirac delta function obeys the relation $\delta(ax) = \frac{1}{|a|}\delta(x)$. (2P)

(b) Likewise prove that $\delta(F(x)) = \sum_i \frac{\delta(x-x_i)}{|F'(x_i)|}$, where the x_i are the (non-degenerate) zeros of the differentiable function F, i.e., $F(x_i) = 0$. (1P)

(c) Let $p_n(x_n)$ be the probability density of x_n, which means that $p_n(x_n)\,\mathrm{d}x_n$ is the probability to find x_n in the infinitesimal interval $[x_n, x_n + \mathrm{d}x_n]$. Later in this lecture we will show that probability density one step later is given by

$$p_{n+1}(x_{n+1}) = \int_{-\infty}^{+\infty} \mathrm{d}x_n\; \delta\Big(x_{n+1} - f(x_n)\Big)\, p_n(x_n).$$

Insert $f(x)$ into this expression and use (b) to simplify it. (2P)

(d) A probability density of a map is called *stationary* if it does not change under iteration, i.e. $p_{n+1}(x) = p_n(x)$. Show that the stationary probability density

$$p_n(x) \propto \frac{1}{\sqrt{x(1-x)}}$$

is a stationary solution. (1P)

Please turn over ⇒

Exercise 2.3: Drunk Driver Statistics (3P)

According to German police statistics in 2009, the age of the drunk drivers involved in car accidents with physical injuries are distributed as follows:

Age	18-24	25-34	35-44	45-54	55-64	$\geq$65
%	26.1	22.4	18.9	18.5	8.7	5.4

At a *given* age, the percentage of *female* drivers involved in such accidents is fairly low:

Age	18-24	25-34	35-44	45-54	55-64	$\geq$65
%	9.1	11.9	15.2	14.4	12.3	9.2

(a) What is probability of being female in an such an accident? (1P)

(b) Compute the probability depending on the age under the condition that the driver is female. Where is the maximum? (2P)

(Σ = 12P)

To be handed in on Monday, October 28, at the beginning of the lecture (PG1 directly to Pascal).