# Statistical Physics & Thermodynamics

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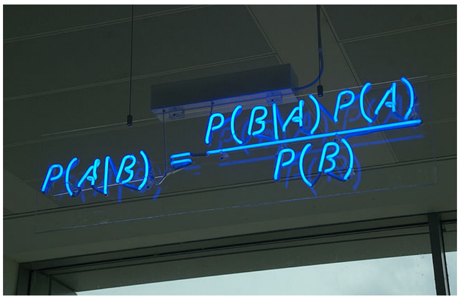
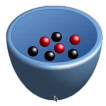


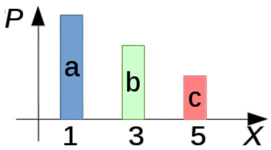

Bayes theorem spelt out in blue neon [Wikimedia]

## Exercise 2.1: Bayes Theorem (2P)



An urn contains three red and four black balls. Two balls are randomly drawn from the urn (without putting them back). What is the probability that the first one is black given that the second one is red?

## Exercise 2.2: Reconstruct probabilities from given moments (4P)




Consider a system with three configurations $\Omega = \{a, b, c\}$ together with an associated map $X : \Omega \to \mathbb{R}$ defined by

$$X_a = 1, \quad X_b = 3, \quad X_c = 5\,.$$

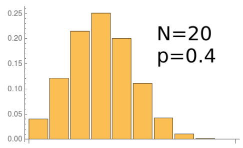
(a) How many moments do you have to know in order to be able to reconstruct the probabilitiy distribution $\{P_a, P_b, P_c\}$ and why?

(b) Suppose that $M_1$ and $M_2$ are given (with the map $X$ defined above). Find the solution for the probabilities $P_a, P_b$, and $P_c$.

(c) Not all possible values for $M_1$ are $M_2$ lead to consistent results. Give a counter-example.

(d) Let $M_1 = \frac{7}{2}$ and $M_2 = 15$. Compute the probabilities $P_a, P_b, P_c$ and find a general formula for the higher moments $M_n$ with $n = 3, 4, \ldots \infty$.

## Exercise 2.3: The binomial distribution (6P)




The binomial distribution reads

$$P_{N,p}(k) = \binom{N}{k} p^k (1-p)^{N-k}\,.$$

(a) Use the relation $\binom{N}{k} = \frac{N}{k}\binom{N-1}{k-1}$ to derive a recursion relation for the (non-centralized) moments $m_n(N,p) = \langle k^n \rangle_{N,p}$ of the binomial distribution. This recursion relation should have the following form: (2P)

$$m_n(N,p) = \text{ some function of } \Big(m_0(N-1,p), m_1(N-1,p), \ldots, m_{n-1}(N-1,p)\Big)$$

(b) Apply this recursion relation to compute the first four non-centralized moments $m_0, \ldots, m_3$ explicitly. (2P)

(c) Compute the moment-generating function $M_{N,p}(t) = \langle e^{kt} \rangle_{N,p}$ of the binomial distribution analytically (please provide a complete proof). (1P)

(d) Compute the first three cumulants $\kappa_0, \ldots, \kappa_3$ explicitly from the cumulant-generating function $K_{N,p}(t) = \ln M_{N,p}(t)$. You are encouraged to use *Mathematica*®or similar computer algebra software. (1P)

**($\Sigma$ = 12P)**

To be handed in on Monday, October 30, at the beginning of the lecture.
We encourage the use of a symbolic computer algebra system such as *Mathematica*®,
but please do not submit uncommented notebook or unreadable output.