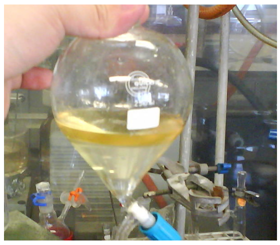
Exercise 12.1: Stability of thermodynamical systems (7P)



Consider a system which exchanges energy and volume with an external reservoir.

(a) Show that the difference of the heat capacities is given by

$$C_p - C_V = T\left(\frac{\partial H}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

Hint: Compare the differentials for $H(T,V)$ and $H(T,p)$ and try to express $\mathrm{d}V$ in terms $\mathrm{d}P$ and $\mathrm{d}T$. Finally substitute the heat capacities $C_{p,V} = T\left(\frac{\partial H}{\partial T}\right)_{p,V}$. (2P)

(b) Verify that $C_p - C_V = \frac{VT\alpha^2}{\kappa_T}$, where $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ is the thermal expansion coefficient and $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$ is the isothermal compressibility. (2P)

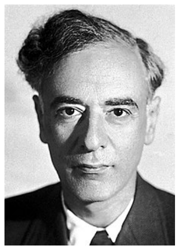
(c) Prove that heat capacities and compressibilities have the same ratio, i.e. $\frac{C_p}{C_V} = \frac{\kappa_T}{\kappa_H}$. (1P)

(d) Use (b), (c) and the properties of κ_T discussed in the lecture to show that the condition of global stability requires the inequalities (1P)

$$C_P \geq C_V \geq 0 \quad \text{and} \quad \kappa_T \geq \kappa_H \geq 0\,.$$

(e) Explain these inequalities on intuitive grounds. (1P)

Exercise 12.2: Landau theory (5P)



Consider a thermodynamical system with the free energy

$$F(\phi,T) \;=\; \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4\,,$$

where α, γ, λ, and T_0 are positive parameters.

(a) Determine the extrema of F as a function of ϕ for given T. (1P)

(b) Show that for $T < T_0$ there is no discontinuous phase transition. (1P)

(c) Prove that for $T > T_0$ the system exhibits a discontinuous phase transition. Compute the corresponding transition temperature $T_c(\alpha,\gamma,\lambda)$. (2P)

(d) Calculate the jump $\Delta\phi$ of the order parameter at $T = T_c$. (1P)

(Σ = 12P)

To be handed in on Monday, January 22, 2018, at the beginning of the lecture.