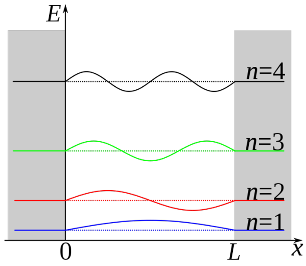


Particle in potential well [Wikimedia]

EXERCISE 9.1: QUANTUM PARTICLE IN A 1D POTENTIAL WELL (8P)

Let us consider a single quantum-mechanical particle in a one-dimensional infinite potential well of length L in contact with a thermal reservoir at temperature T.

(a) Compute the energy eigenvalues and write down the partition sum. (1P)

(b) Show that in the low-temperature limit $T \to 0$ the logarithm of the partition sum is given by leading and next-to-leading order by (2P)

$$\ln Z(\beta, L) = -\frac{\beta\gamma}{L^2} + e^{-3\frac{\beta\gamma}{L^2}} + \mathcal{O}\big(e^{-8\frac{\beta\gamma}{L^2}}\big) \quad \text{where} \quad \gamma = \frac{\pi^2\hbar^2}{2m}.$$

(c) Compute the average energy E, the heat capacity C, and the pressure P in the limit of low temperatures by calculating the corresponding derivatives of $\ln Z$. (1P)

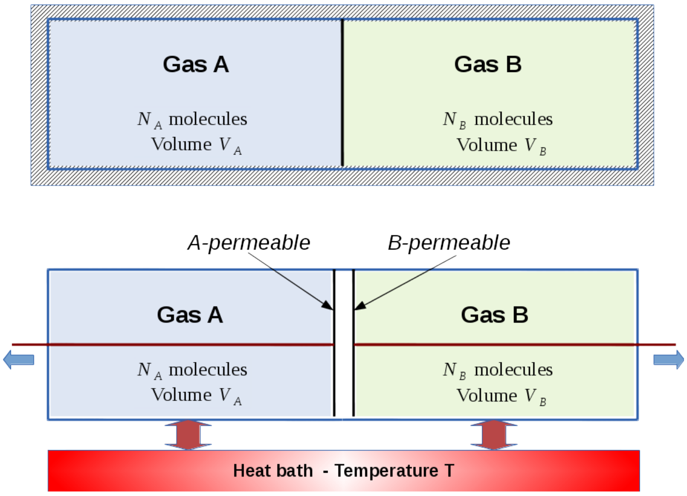
(d) Why is $P > 0$ in the limit $T \to 0$? (1P)

(e) Approximate the partition sum in the high-temperature limit $T \to \infty$. (2P)

(f) Compute E, C, and P in the limit of high temperatures. (1P)

EXERCISE 9.2: MIXING TWO GASES (4P)

Let us consider two possibilities to mix two gases. In the first case an isolated container is divided into two chambers containing pure gases of two different types A and B, which are mixed by removing the separating wall in the middle. In the second case the container is connected to a heat bath of temperature T and the two gases are initially separated by two membranes which are permeable either exclusively to A or to B-particles. The mixing is carried out by moving the membranes quasi-statically to the left and to the right, as indicated in the figure.



Please turn over ⇒

(a) Compute the change of system entropy in the first scenario. (1P)

(b) Compute the change of system entropy in the second scenario by relating it to the work needed to pull the membranes. The gases are assumed to be ideal, obeying the equation of state $pV = Nk_BT$. (2P)

(c) Determine the entropy change in the heat bath in the second scenario. (1P)

(Σ = 12P)

To be handed in on Monday, December 18, at the beginning of the lecture.